# Examination of Acidity Functions by Characteristic Vector Analysis $\boldsymbol{\dagger}$ 

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#### Abstract

Characteristic vector analysis has been applied to 112 acidity function data sets of various strong acids as variables of the percentage, and to 51 acidity function data sets as variables of the molar concentration in water-organic solvents. It has been found that only one characteristic vector, $V$, an independent variable, produces differences in each set of acidity functions. This vector is a statistically universal measure of acidity and depends on acid concentration. It is independent of the nature of the acid. A further parameter, $S$, depends on the nature of the acid and on the acidity function. The reconstituted acidity function given by the equation $H_{m, t}=A_{m, l}+V_{1, t} S_{1, m}$ explains more than $98 \%$ of the experimental variations of the acidity function.


The acidity function concept was developed by Hammett ${ }^{1}$ who defined the acidity function $H_{0}$ for sulphuric acid solutions as an extension of the pH scale into the concentrated acid region. It become apparent, however, after experimental work by Arnett ${ }^{2}$ and others ${ }^{3-5}$ that $H_{0}$ was not a unique function but only one of many possible acidity functions. Numerous acidity functions have been developed for water and for water-organic solvents by application of the classical Hammett postulate concerning the activity coefficient term and experimental procedure. The ' failure ' of acidity functions was discussed in terms of indicator structure, hydration phenomena, salt effects, ${ }^{2}$ and their influence on the activity coefficient term.

In 1966 Bunnett and Olsen showed that another assumption regarding the activity coefficient term leads to considerable simplification ${ }^{6}$ and the number of different acidity functions needed to explain the protonation of different bases has been reduced to one. The linear relations between various acidity functions were presented ${ }^{7}$ as an evidence of the validity of the Hammett postulate (or cancellation assumption). The Ham-mett-type acidity function is still needed and to derive it, two assumptions are involved; these are cancellation and linearity, the latter being more important. This is a method used by Marziano and her co-workers ${ }^{8}$ for acid systems and by Cox and his co-workers ${ }^{9}$ for basic media. These groups tested the validity of the linearity assumption. Hence the assertion that there is only one acidity function per acid system. New, more general acidity scales were exemplified by $M_{c}{ }^{8}$ and the activity coefficient ' excess' acidity function $X .^{10}$

A different approach to acidity function ' failure ' was used by Palm and his co-workers. ${ }^{11}$ Differences between acidity functions were explained by a four-parameter equation involving polarity, polarizibility, Lewis acidity, and basicity; however the results were far from satisfactory.

This controversy involving the acidity function problem is of essential importance for physical organic chemistry ${ }^{12}$ because of the need to study the protonation equilibrium constant, $\mathrm{p} \mathrm{K}_{\mathrm{BH}^{+}}$and the relation between reaction rates and medium acidity in acid-catalysed reactions.

This paper attempts a more direct approach to the acidity function problem. This approach is based on characteristic vector analysis (c.v.a.) ${ }^{13}$ of existing acidity function data. The power of this approach is that it is not based on a specific

[^0]theory relating acidity function data to the structure of organic bases and/or type of acid. Rather, the approach is based on a general theory of the behaviour of data observed for similar processes-protonations in strong acid medium. The mathematical theory is in fact an extension of the familiar concept of linear free energy relationship and makes it possible to arrive at reliable conclusions provided the data have been selected in a manner adequate to the problem under study.

Design of the Study.-C.v.a. is a very useful mathematical method to deal with the problem of seeking regularities in chemical or physical data. Recently c.v.a. or a similar approach was used to study intramolecular interactions in the liquid state, ${ }^{14}$ kinetic data for solvolytic reactions, ${ }^{15}$ extraction of mass spectra from g.l.c.-m.s. data of unseparated mixtures, ${ }^{16}$ and protonation phenomena by u.v. spectroscopy. ${ }^{17}$

We have studied processes in the form of acidity function sets as variables of percentage or molarity. Two types of acidity functions are of interest, those established according to the classical procedure ${ }^{1}$ and described by equation (1) and those characterizing acids in a simple way (e.g. the activity or

$$
\begin{equation*}
H=-\log a_{\mathbf{H}^{+}} \cdot \gamma_{\mathrm{B}} / \gamma_{\mathrm{BH}^{+}} \tag{1}
\end{equation*}
$$

concentration of species existing in acids). All acidity function data were from the literature.

Acidity Functions as Variables of Percentage.-We utilized 41 acidity functions for sulphuric acid, 25 for perchloric acid, 13 for phosphoric acid, eight for nitric acid, seven for hydrochloric acid, six for toluene-p-sulphonic acid, and 12 for miscellaneous acids. The types of acidity functions and references are collected in Table 1.
The values of the acidity functions at the required percentages ( $5 \%$ and multiples) were used directly from original source, if available, or were read from the graphs of acidity functions versus percentages at desired concentrations.
In spite of employing acid concentration ranges characteristic of particular acids the data matrix was not complete along the concentration axis. The concentration range of hydrochloric acid was $5-37 \%$ and after a short extrapolation the last reading was at $40 \%$. Most acidity functions for sulphuric acid cover the range 5-95\% but some data were available only in the range up to $40 \%$. A similar situation applies to the remaining acids. Because of the requirements of the c.v.a. method (a complete matrix) the initial incomplete matrix was divided into several submatrices taking the percentage of the acid concentration as the criterion. This division was a compromise between unnecessary omission of existing data, un-

Table 1. Acidity functions, ranges of acid concentration (\%), and values of scalars $S_{1}$ and $S_{2}$

| Acidity function ${ }^{\text {a }}$ | Concentration range ${ }^{b}$ | $S_{1}$ | $S_{2}$ | $c$ | Ref. | Acidity function ${ }^{a}$ | Concentration range ${ }^{b}$ | $S_{1}$ | $S_{2}$ | $c$ | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 \mathrm{~S} \mathrm{H}_{0}$ | xxxxxx | -0.0891 | -0.0254 | A | 18 | $29 \mathrm{~S} \mathrm{H}_{0}{ }^{1}$ | xxxxxx | -0.3178 | -0.1956 | A | 32 |
| $2 \mathrm{~S} \mathrm{H}_{0} 25$ | xxxxxx | -0.0900 | 0.0258 | A | 19 | $30 \mathrm{~S} \mathrm{H}_{0}{ }^{2}$ | xxxxxx | -0.2026 | -0.0925 | A | 32 |
| $3 \mathrm{~S} \mathrm{H}_{0} 40$ | xxxxx | -0.0729 | -0.0033 | A | 19 | $31 \mathrm{~S} \mathrm{H}_{0}{ }^{3}$ | xxxxx | -0.0867 | -0.0124 | A | 32 |
| $4 \mathrm{~S} H_{0} 60$ | xxxxxx | -0.0538 | 0.0077 | A | 19 | $32 \mathrm{~S} H_{0}{ }^{4}$ | Xxxx | -0.0602 | -0.3782 | D | 32 |
| $5 \mathrm{~S} \mathrm{H}_{0} 80$ | xxxxxx | -0.0415 | -0.0162 | A | 19 | $33 \mathrm{~S} H_{0} \mathrm{DMSO}$ | xxxxxx | -0.2832 | 0.2438 | B | 33 |
| $6 \mathrm{~S} \mathrm{H}_{0} 90$ | xxxxxx | -0.0264 | -0.0353 | A | 19 | 34 S $H_{0}{ }^{\text {D }}$ DMSO | xxxxxx | -0.3727 | 0.0375 | B | 33 |
| $7 \mathrm{~S} \mathrm{H} \mathrm{H}_{\mathrm{R}} 5$ | xxxxx | -0.4558 | 0.0918 | C | 20 | 35 S $H_{0}{ }^{+} \lg C_{\text {A }}$ | xxxxxx | -0.0248 | 0.2624 | A | 34 |
| $8 \mathrm{~S} H_{\mathrm{R}} 40$ | xxxxx | -0.4023 | 0.0335 | C | 20 | $36 \mathrm{~S} \lg C_{\mathrm{A}}$ | xxxxxx | 0.5323 | -0.2185 | B | 34 |
| $9 \mathrm{~S} H_{\mathrm{R}} 60$ | xxxxx | -0.3070 | -0.0685 | C | 20 | $37 \mathrm{~S} \lg a_{w}$ | xxxxxx | 0.1719 | 0.6555 | A | 35 |
| $10 \mathrm{~S} H_{\mathrm{R}} 80$ | xxxxx | -0.1948 | -0.0698 | C | 20 | $38 \mathrm{~S} \lg C_{\mathrm{H}}$ | xxxxxx | 0.5275 | -0.0299 | B | 10 |
| $11 \mathrm{~S} H_{\mathrm{R}} 90$ | xxxxx | -0.1395 | -0.1165 | C | 20 | $39 \mathrm{~S} \lg a_{\mathrm{H}}$ | xxxx | 0.6040 | -0.1194 | D | 36 |
| $12 \mathrm{~S} H_{\mathrm{x}}$ | xxxxx | 0.6090 | -0.5610 | A | 10 | 40 S $\lg a_{w}+$ | xxxx | 0.5387 | 0.0751 | D | 36 |
| $13 \mathrm{~S} H_{\mathrm{A}}$ | xxxxx | 0.1076 | -0.3295 | B | 3 | $41 \mathrm{~S} \lg C_{\mathbf{H}}{ }^{*}$ | xx | 0.3581 | 0.0090 | F | 37 |
| $14 \mathrm{~S} H_{\mathrm{A}}$ | xxxxx | 0.1392 | -0.3925 | C | 21 | $42 \mathrm{C} H_{0}$ | xxx | -0.1756 | 0.5023 | E | 38 |
| $15 \mathrm{~S} H_{\mathrm{A}}$ | x | 0.2488 | 0.0140 | G | 22 | $43 \mathrm{CH} H_{\mathrm{R}}$ | xx | -0.2564 | 0.0012 | F | 39 |
| $16 \mathrm{~S} \mathrm{H} H^{\prime \prime}{ }_{0}$ | xxxxxx | -0.2192 | -0.0281 | B | 2 | $44 \mathrm{C} H_{\mathrm{x}}$ | xxx | 0.5934 | -0.2452 | E | 10 |
| $17 \mathrm{~S} \mathrm{H}^{\prime \prime}{ }_{0}$ | xxx | -0.1080 | -0.0504 | E | 23 | $45 \mathrm{C} H_{\mathrm{A}}$ | xxx | -0.0366 | -0.1513 | E | 40 |
| $18 \mathrm{~S} \mathrm{H}{ }_{0}{ }^{\text {a }}$ | xxx | -0.1079 | -0.0543 | E | 23 | $46 \mathrm{C} H_{\mathrm{A}} 15$ | xx | -0.0304 | -0.0500 | F | 41 |
| $19 \mathrm{~S} H^{\prime}$ | xxxxxx | -0.3798 | -0.4401 | B | 24 | $47 \mathrm{C} H_{\mathrm{A}} 25$ | x $x$ | -0.0392 | -0.0524 | F | 41 |
| $20 \mathrm{~S} H_{1}$ | xxx | -0.2409 | -0.2015 | E | 4 | $48 \mathrm{C} \mathrm{H}_{\mathrm{A}} 35$ | xx | -0.0281 | -0.0566 | F | 41 |
| $21 \mathrm{SH} H_{M}$ | xxx | -0.2816 | -0.3661 | E | 25 | $49 \mathrm{C} H_{\mathrm{A}} 45$ | xx | -0.0267 | -0.0633 | F | 41 |
| $22 \mathrm{~S} H_{\mathrm{c}}$ | xxxx | -0.3281 | -0.4775 | D | 26 | $50 \mathrm{C} \boldsymbol{H}^{\prime \prime \prime}{ }_{0}$ | xxx | -0.3428 | 0.4210 | E | 40 |
| 23 S Mc | xxxxxx | 0.2570 | -0.0102 | A | 27 | $51 \mathrm{C} H_{1}$ | x | 0.604 | 0.2348 | C | 4 |
| $24 \mathrm{~S} \mathrm{H} \mathrm{HFF}^{\text {c }}$ | xx | -0.1443 | 0.0342 | F | 28 | 52 CH H | xx | -0.1987 | 0.0799 | F | 42 |
| $25 \mathrm{~S} H^{+}$ | xxx | -0.0988 | -0.3478 | E | 29 | $53 \mathrm{C} H_{\text {c }}$ | xx | -0.1772 | 0.1315 | F | 26 |
| $26 \mathrm{~S} H_{\mathrm{T}}$ | xxxx | -0.2667 | -0.3664 | D | 30 | $54 \mathrm{C} \Delta$ | xx | 0.1577 | 0.0251 | F | 21 |
| 27 S $\Delta$ | xxxxxx | 0.4088 | 0.0682 | A | 31 | $55 \mathrm{C} \mathrm{H}_{0}{ }^{0}$ | xx | -0.2432 | 0.0626 | F | 32 |
| $28 \mathrm{~S} \mathrm{H}_{0}{ }^{\text {o }}$ | xxxxxx | -0.4352 | -0.2991 | A | 32 | $56 \mathrm{C} H_{0}{ }^{1}$ | xx | -0.1825 | 0.0660 | F | 32 |


| Acidity function ${ }^{a}$ | Concentration range ${ }^{b}$ | $S_{1}$ | $S_{2}$ | $c$ | Ref. | Acidity function ${ }^{a}$ | Concentration range ${ }^{b}$ | $S_{1}$ | $S_{2}$ | $c$ | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $57 \mathrm{C} \mathrm{H}_{0}{ }^{2}$ | xx | -0.1219 | 0.0587 | F | 32 | 85 TS $H_{\text {GF }}$ | xx | 0.0513 | 0.1703 | F | 46 |
| $58 \mathrm{CH} \mathrm{H}^{3}$ | xX | -0.0611 | 0.0587 | F | 32 | 86 TS $\lg C_{\text {H }}{ }^{*}$ | x ${ }^{\text {x }}$ | 0.2112 | 0.2514 | F | 46 |
| $59 \mathrm{C} \mathrm{H}_{0}{ }^{\text {a }}$ | xx | 0.0003 | 0.0600 | F | 32 | $87 \mathrm{~N} \mathrm{H}_{0}$ | x | 0.2640 | -0.1794 | G | 39 |
| $60 \mathrm{C} H_{0}{ }^{5}$ | xx | 0.0603 | 0.0561 | F | 32 | $88 \mathrm{NH}_{\mathrm{R}}$ | xx | -0.2072 | -0.3304 | F | 39 |
| $61 \mathrm{C} D_{0}$ | xx | -0.0832 | 0.0996 | F | 43 | 89 Na | xx | 0.1466 | 0.0491 | F | 32 |
| $62 \mathrm{C} D_{\mathrm{x}}$ | xx | -0.0410 | 0.1317 | F | 43 | $90 \mathrm{~N} \mathrm{H}_{0}{ }^{0}$ | xx | -0.2072 | -0.3305 | F | 32 |
| $63 \mathrm{C} \mathrm{H}^{\prime}{ }_{0} \mathrm{EtOH}$ | x | 0.1311 | 0.7753 | G | 44 | $91 \mathrm{~N} H_{0}{ }^{1}$ | xx | -0.1646 | -0.1896 | F | 32 |
| $64 \mathrm{Clg} C_{\mathrm{H}}$ | xxx | 0.3373 | 0.4625 | E | 10 | $92 \mathrm{NH}_{0}{ }^{2}$ | xx | -0.0989 | -0.2403 | F | 32 |
| $65 \mathrm{Clg} a_{\mathrm{H}}$ | xx | 0.4083 | -0.0801 | F | 36 | $93 \mathrm{NH}{ }_{0}{ }^{3}$ | xx | -0.0440 | -0.1935 | F | 32 |
| $66 \mathrm{C} \lg a_{\mathrm{w}}+$ | xx | 0.1119 | -0.0497 | F | 36 | $94 \mathrm{~N} \mathrm{H}_{0}{ }^{4}$ | xx | 0.0095 | -0.1492 | F | 32 |
| $67 \mathrm{P} H_{\mathrm{R}}$ | xxxx | -0.1727 | 0.2709 | D | 2 | $95 \mathrm{Cl} H_{0}$ | x | -0.0503 | 0.1718 | G | 45 |
| $68 \mathrm{P} H^{\prime \prime \prime}{ }_{0}$ | xxxx | -0.0599 | 0.0976 | D | 2 | $96 \mathrm{Cl} H_{\mathrm{R}}$ | x | -0.5066 | 0.0688 | G | 46 |
| $69 \mathrm{P} H^{\prime}{ }_{0}$ | xxxx | 0.0241 | 0.1446 | D | 2 | $97 \mathrm{Cl} \mathrm{H}_{\mathrm{A}}$ | x | 0.0012 | -0.3119 | G | 47 |
| $70 \mathrm{P} \mathrm{H}_{\mathrm{R}}^{\prime}$ | xxxx | -0.1308 | 0.2053 | D | 2 | $98 \mathrm{Cl} \mathrm{H}{ }_{0}$ | x | -0.0544 | -0.1078 | G | 2 |
| $71 \mathrm{P} H_{\text {GF }}$ | xxxx | -0.1203 | 0.1529 | D | 37 | $99 \mathrm{Cl} \mathrm{H}^{\prime \prime \prime}{ }_{0}$ | x | -0.2519 | -0.2967 | G | 2 |
| $72 \mathrm{Plg} \mathrm{C}_{\mathrm{H}}{ }^{*}$ | xx | 0.2407 | -0.1435 | F | 37 | $100 \mathrm{Cl} \mathrm{H} \mathrm{H}_{\text {R }}$ | x | -0.4259 | -0.1034 | G | 2 |
| $73 \mathrm{P} \mathrm{H}^{\prime}{ }^{+} \lg a_{\mathbf{H}}+$ | xx | 0.1978 | 0.1237 | F | - | $101 \mathrm{Cl} \Delta$ | x | 0.5836 | -0.2635 | G | 31 |
| $74 \mathrm{P} \Delta$ | xxxx | 0.1989 | -0.3898 | D | 32 | $102 \mathrm{~F} H_{0}$ | xx | -0.0612 | 0.4491 | F | 48 |
| $75 \mathrm{P} \mathrm{H}_{0}{ }^{0}$ | xxxx | -0.1729 | 0.2759 | D | 32 | $103 \mathrm{~F} \Delta$ | xx | 0.1436 | -0.1078 | F | 49 |
| $76 \mathrm{P} \mathrm{H}_{0}{ }^{1}$ | xxxx | -0.1070 | 0.2049 | D | 32 | $104 \mathrm{FF} H_{0}{ }^{3}$ | xx | -0.0612 | 0.3923 | F | 49 |
| $77 \mathrm{P} \mathrm{H}_{0}{ }^{2}$ | xxxx | -0.0414 | 0.1520 | D | 32 | $105 \mathrm{MS} \lg C_{\mathbf{H}}{ }^{*}$ | xx | 0.2408 | 0.0685 | F | 37 |
| $78 \mathrm{P} \mathrm{H}_{0}{ }^{3}$ | xxxx | 0.0243 | 0.1043 | D | 32 | 106 MS $H_{\text {GF }}$ | xxxxx | 0.0694 | -0.1279 | C | 37 |
| $79 \mathrm{P} \mathrm{H0}{ }^{4}$ | xxxx | 0.0902 | 0.0475 | D | 32 | $107 \mathrm{CF}_{3} H_{\mathrm{GF}}$ | xxxxxx | 0.0954 | 0.7626 | B | 38 |
| $80 \mathrm{HP} \lg C_{\mathrm{H}+}+$ | xxx | 0.4629 | 0.0667 | E | 37 | 108 SE $H_{0}$ | xxxxxx | 0.0058 | 0.2066 | A | 49 |
| 81 TS | xx | 0.0595 | 0.1126 | F | 2 | 109 HC $\Delta$ | xxxxx | 0.4673 | -0.5570 | C | 32 |
| $82 \mathrm{TS} H_{\mathrm{R}}$ | xx | 0.0042 | -0.0385 | F | 2 | $110 \mathrm{HC} \mathrm{H}_{0}{ }^{0}$ | xxxxx | 0.2030 | 0.4228 | C | 32 |
| 83 TS $H^{\prime \prime \prime}{ }_{0}$ | xx | 0.0589 | 0.1490 | F | 2 | $111 \mathrm{HC} H_{0}{ }^{1}$ | xxxxx | 0.2744 | 0.4039 | C | 32 |
| 84 TS HR | xx | 0.0079 | -0.0450 | F | 2 | $112 \mathrm{HC} \mathrm{H}_{0}$ | xxxxx | 0.3548 | 0.3804 | C | 32 |

${ }^{\text {a }}$ S, Sulphuric acid; C, perchloric acid; P, phosphoric acid; HP, phosphonic acid; TS, toluene-p-sulphonic acid; N, nitric acid; Cl, hydrochloric acid; B, hydrobromic acid; J, iodic acid; F, hydrofluoric acid; MS, methane sulphonic acid; FS, fluorosulphonic acid; $\mathrm{CF}_{3}$ trifluoroacetic acid; SE, selenic acid; HC, formic acid; Et, ethanol; DX, dioxan; Be, 2-butoxyethanol; AN, acetonitrile. ${ }^{b} \mathrm{x}, 40 \%$; xx, $55 \%$; xxx, $70 \%$; xxxx, $80 \%$, xxxxx, $90 \%$; xxxxxx, $95 \%{ }^{c}{ }^{c}$ Submatrix A-G.
necessary extrapolation of non-existing data, and dealing with a reasonably small number of submatrices. Only in a very few cases was extrapolation done over a range of $<3 \%$, and only few data were neglected.

The following ranges of acid concentration were examined, $5-40,5-55,5-70,5-80,5-90$, and $5-95 \%$, and are indicated in Table 1 for each acidity function by $G, F, E, D$, $C$, and $A / B$, respectively.

Table 2. Results of c.v.a. calculation for submatrices A-G (percentage mode)

| Submatrix | $\begin{gathered} \text { \%Total } \\ \text { variation } \end{gathered}$ |  | $N^{a}$ |  | Concentration (\%) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $V_{1}$ | $V_{1}+V_{2}$ |  |  | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| $\begin{aligned} & 95 \% \\ & \mathrm{~A} \end{aligned}$ | 99.82 | 99.98 |  | $V_{1}$ | 0.066 | 0.927 | 1.723 | 2.543 | 3.399 | 4.301 | 5.311 | 6.371 | 7.510 | 8.788 |
|  |  |  | 16 | $V_{2}$ | -0.042 | 0.138 | 0.288 | 0.391 | 0.494 | 0.572 | 0.683 | 0.766 | 0.813 | 0.858 |
|  |  |  |  | $-\bar{H}$ | 0.059 | -0.250 | 0.509 | 0.774 | 1.029 | 1.293 | 1.564 | 1.854 | 2.176 | 2.545 |
| 95\% | 99.22 | 99.85 | 8 | $V_{1}$ | -0.229 | 0.867 | 1.668 | 2.411 | 3.137 | 3.842 | 4.608 | 5.475 | 6.437 | 7.403 |
| B |  |  |  | $V_{2}$ | 0.723 | 0.737 | 0.724 | 0.747 | 0.755 | 0.789 | 0.808 | 0.793 | 0.714 | 0.640 |
|  |  |  |  | $-\bar{H}$ | -0.123 | 0.233 | 0.532 | 0.817 | 1.108 | 1.403 | 1.732 | 2.113 | 2.542 | 2.982 |
| $\begin{aligned} & 90 \% \\ & \mathrm{C} \end{aligned}$ | 98.94 | 99.83 | 11 | $V_{1}$ | 1.302 | 1.756 | 2.533 | 3.141 | 2.843 | 4.645 | 5.453 | 6.448 | 7.483 | 8.504 |
|  |  |  |  | $V_{2}$ | 1.790 | 1.766 | 1.653 | 1.513 | 1.331 | 1.213 | 0.983 | 0.763 | 0.767 | 0.357 |
|  |  |  |  | $-\bar{H}$ | -0.527 | -0.201 | 0.193 | 0.502 | 0.884 | 1.285 | 1.664 | 2.093 | 2.575 | 3.061 |
| $\begin{aligned} & 80 \% \\ & \mathrm{D} \end{aligned}$ | 99.57 | 99.93 | 16 | $V_{1}$ | -1.016 | 0.335 | 1.662 | 2.877 | 4.149 | 5.445 | 6.822 | 8.406 | 10.18 | 12.02 |
|  |  |  |  | $V_{2}$ | 1.468 | 1.337 | 1.159 | 1.003 | 0.873 | 0.716 | 0.589 | 0.547 | 0.519 | 0.468 |
|  |  |  |  | $\bar{H}$ | -0.607 | -0.377 | -0.165 | 0.058 | 0.272 | 0.480 | 0.689 | 0.903 | 1.149 | 1.428 |
| 70\% | 99.26 | 99.94 | 11 | $V_{1}$ | -0.271 | 0.837 | 1.742 | 2.565 | 3.357 | 4.264 | 5.259 | 6.408 | 7.641 | 8.680 |
| E |  |  |  | $V_{2}$ | -0.146 | 0.193 | 0.405 | 0.540 | 0.671 | 0.710 | 0.788 | 0.860 | 0.866 | 0.787 |
|  |  |  |  | $-\bar{H}$ | 0.012 | 0.300 | 0.536 | 0.749 | 0.968 | 1.187 | 1.432 | 1.705 | 1.996 | 2.285 |
| 55\% | 98.82 | 99.83 | 39 | $V_{1}$ | 0.130 | 2.641 | 4.929 | 6.963 | 8.951 | 11.05 | 13.37 | 16.05 | 19.02 | 22.45 |
| F |  |  |  | $V_{2}$ | 1.642 | 1.611 | 1.582 | 1.565 | 1.538 | 1.475 | 1.271 | 0.818 | 0.037 | -1.102 |
|  |  |  |  | $-\bar{H}$ | -0.011 | 0.244 | 0.458 | 0.654 | 0.849 | 1.045 | 1.245 | 1.479 | 1.742 | 2.061 |
| 40\% | 99.42 | 99.82 | 11 | $V]$ |  | 1.890 | 2.818 | 3.852 | 5.026 | 6.294 | 7.495 | 8.719 |  |  |
| G |  |  |  | $V_{2}$ |  | 0.601 | 0.600 | 0.352 | 0.243 | 0.003 | -0.192 | -0.453 |  |  |
|  |  |  |  | $-\bar{H}$ |  | 0.844 | 1.344 | 1.896 | 2.441 | 3.040 | 3.637 | 4.213 |  |  |


|  |  |  |  |  | Concentration (\%) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| matrix | $\overparen{V_{1}}$ | $V_{1}+V_{2}$ | $N^{a}$ |  | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 |
| $\begin{aligned} & 95 \% \\ & \mathrm{~A} \end{aligned}$ | 99.82 | 99.98 | 16 | $\begin{array}{r} V_{1} \\ -\overline{V_{2}} \\ -\bar{H} \end{array}$ | $\begin{gathered} 10.21 \\ 0.903 \\ 2.949 \end{gathered}$ | $\begin{gathered} 11.84 \\ 0.960 \\ 3.410 \end{gathered}$ | $\begin{gathered} 13.63 \\ 0.946 \\ 3.931 \end{gathered}$ | $\begin{gathered} 15.47 \\ 0.785 \\ 4.474 \end{gathered}$ | $\begin{gathered} 17.42 \\ 0.468 \\ 5.068 \end{gathered}$ | $\begin{gathered} 19.35 \\ -0.020 \\ 5.647 \end{gathered}$ | $\begin{array}{r} 21.47 \\ -0.440 \\ 6.284 \end{array}$ | $\begin{array}{r} 23.39 \\ -1.046 \\ 6.862 \end{array}$ | $\begin{array}{r} 25.50 \\ -1.862 \\ 7.568 \end{array}$ |
| $\begin{gathered} 95 \% \\ \text { B } \end{gathered}$ | 99.22 | 99.85 | 8 | $\begin{array}{r} V_{1} \\ -\overline{V_{2}} \\ -\bar{H} \end{array}$ | $\begin{aligned} & 8.363 \\ & 0.594 \\ & 3.429 \end{aligned}$ | $\begin{aligned} & 9.518 \\ & 0.616 \\ & 3.968 \end{aligned}$ | $\begin{gathered} 10.81 \\ 0.524 \\ 4.557 \end{gathered}$ | $\begin{gathered} 12.05 \\ 0.459 \\ 5.158 \end{gathered}$ | $\begin{gathered} 13.13 \\ 0.295 \\ 5.718 \end{gathered}$ | $\begin{array}{r} 14.04 \\ 0.157 \\ 8.242 \end{array}$ | $\begin{array}{r} 15.26 \\ -0.590 \\ 8.969 \end{array}$ | $\begin{array}{r} 16.03 \\ -1.176 \\ 7.578 \end{array}$ | $\begin{array}{r} 16.64 \\ -1.595 \\ 8.123 \end{array}$ |
| $90 \%$ | 98.94 | 99.83 | 11 | $\begin{array}{r} V_{1} \\ -\overline{V_{2}} \\ -\bar{H} \end{array}$ | $\begin{aligned} & 9.797 \\ & 0.195 \\ & 3.608 \end{aligned}$ | $\begin{gathered} 11.25 \\ 0.060 \\ 4.223 \end{gathered}$ | $\begin{array}{r} 12.47 \\ -0.067 \\ 4.888 \end{array}$ | $\begin{array}{r} 14.06 \\ -0.193 \\ 5.523 \end{array}$ | $\begin{array}{r} 15.48 \\ -0.312 \\ 6.208 \end{array}$ | $\begin{array}{r} 16.44 \\ -0.466 \\ 6.808 \end{array}$ | $\begin{array}{r} 17.62 \\ -0.717 \\ 7.498 \end{array}$ | $\begin{gathered} 18.74 \\ -0.882 \\ 8.239 \end{gathered}$ |  |
| $\begin{aligned} & 80 \% \\ & \mathrm{D} \end{aligned}$ | 99.57 | 99.93 | 16 | $\begin{array}{r} V_{1} \\ -V_{2} \\ -\bar{H} \end{array}$ | $\begin{gathered} 13.85 \\ 0.385 \\ 1.708 \end{gathered}$ | $\begin{gathered} 15.82 \\ 0.220 \\ 2.024 \end{gathered}$ | $\begin{array}{r} 17.91 \\ -0.136 \\ 2.386 \end{array}$ | $\begin{array}{r} 20.26 \\ -0.291 \\ 2.765 \end{array}$ | $\begin{array}{r} 22.25 \\ -0.511 \\ 3.213 \end{array}$ | $\begin{gathered} 24.92 \\ -0.809 \\ 3.621 \end{gathered}$ |  |  |  |
| $\begin{aligned} & 70 \% \\ & \mathrm{E} \end{aligned}$ | 99.26 | 99.94 | 11 | $\begin{array}{r} V_{1} \\ -\overline{V_{2}} \end{array}$ | $\begin{gathered} 10.57 \\ 0.437 \\ 2.657 \end{gathered}$ | $\begin{array}{r} 12.42 \\ -0.081 \\ 3.075 \end{array}$ | $\begin{array}{r} 14.52 \\ -0.742 \\ 3.537 \end{array}$ | $\begin{gathered} 16.79 \\ -1.416 \\ 4.0494 \end{gathered}$ |  |  |  |  |  |
| $\begin{aligned} & 55 \% \\ & \mathrm{~F} \end{aligned}$ | 98.82 | 99.83 | 39 | $\begin{array}{r} V_{1} \\ -\overline{V_{2}} \\ -\bar{H} \end{array}$ | $\begin{array}{r} 26.60 \\ -2.231 \\ 2.454 \end{array}$ |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 40 \% \\ & G \end{aligned}$ | 99.42 | 99.82 | 11 | $\begin{array}{r} V_{1} \\ -\overline{V_{2}} \\ -\bar{H} \end{array}$ |  |  |  |  |  |  |  |  |  |

${ }^{a}$ Number of acidity functions.

Acidity Functions as Variable of Molarity.-We utilize 24 acidity functions for sulphuric, perchloric, and hydrochloric acids in water-organic solvents. In addition six acidity functions for less popular acids were studied. In all cases there were no data available for conversion of the molar concentrations into percentages. We also included 21 of the most important
acidity functions for sulphuric, perchloric, and hydrochloric acids in water.

A types of acidity functions and references to the original source were collected in Table 2, together with molarity range. An initial and incomplete data matrix was divided into several submatrices taking molarity of acids as criterion. The following

Table 3. Acidity functions, ranges of molar concentration, and values of $S_{1}$ and $S_{2}$

|  | Acidity function | Concentration range ${ }^{b}$ | $S_{m, 1}$ | $S_{m .2}$ | $c$ | Ref. | $S_{m, 1} 4 \mathrm{~m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | S $H_{0}$ | xxxxx | -0.0474 | -0.1279 | H | 18 | -0.0471 |
| 2 | S $\mathrm{H}_{0} \mathrm{Et} 20 \%$ | xxxxx | -0.0863 | 0.0452 | H | 50 | -0.0416 |
| 3 | S $H_{0} \mathrm{Et} 20 \%$ | xxxxx | -0.0838 | 0.0193 | H | 51 | -0.0522 |
| 4 | S $H^{\prime} \mathrm{Et} 20 \%$ | xxxxx | -0.1655 | 0.0423 | H | 52 | -0.0942 |
| 5 | S $H_{0} \mathrm{Et} 50 \%$ | xxxx | -0.1449 | -0.1801 | I | 53 | -0.1242 |
| 6 | S $H_{\mathrm{R}}$ | xxxxx | -0.2808 | -0.0012 | H | 20 | -0.1675 |
| 7 | S ${ }^{0} H_{0}$ | xxxxx | -0.2570 | -0.1382 | H | 47 |  |
| 8 | S ${ }^{0} H_{0} \mathrm{Et20} \%$ | xxxxx | -0.3366 | 0.0359 | H | 47 | -0.2095 |
| 9 | S $H_{\text {GF }}$ | xxxxx | -0.1772 | -0.1264 | H | 28 | -0.1261 |
| 10 | S $H_{\text {GF }} \mathrm{Et} 20 \%$ | xxxxx | -0.1217 | 0.7911 | H | 28 | 0.0433 |
| 11 | S $H_{\text {GF }} \mathrm{DX} 40 \%$ | xxxx | -0.0714 | 0.8785 | I | 28 | 0.0184 |
| 12 | S $H_{\text {GF }} \mathrm{DX} 60 \%$ | xxx | -0.8001 | 0.1627 | K | 28 | 0.0853 |
| 13 | S $H_{\text {GF }} \mathrm{AN} 75 \%$ | xx | -0.4296 | 0.4506 | L | 28 |  |
| 14 | S $\Delta$ | xxxxx | 0.2418 | -0.0019 | H | 31 | 0.1568 |
| 15 | S $\triangle$ Et $20 \%$ | xxxxx | 0.2538 | -0.0159 | H | 47 | 0.1636 |
| 16 | S $\lg a_{\mathrm{H}}$ | xxxxx | 0.5588 | 0.1173 | H | 37 | 0.3752 |
| 17 | S $M_{\text {c }}$ | xxxxxx | -0.0038 | 0.0837 | H | 27 | 0.0087 |
| 18 | S $H_{\mathrm{x}}$ | xxxx | 0.3219 | -0.0324 | H | 10 | 0.2027 |
| 19 | $\mathrm{S} H_{\mathrm{A}}$ | xxxxx | 0.0072 | -0.2435 | H | 3 | -0.0275 |
| 20 | $\mathrm{S} \boldsymbol{H}_{+}$ | xxxxx | -0.0499 | -0.2509 | H | 29 | -0.0663 |
| 21 | C $\mathrm{H}_{0}$ | xxxxx | -0.0748 | 0.0734 | H | 38 |  |
| 22 | C $\mathrm{H}_{0} \mathrm{Et50} \mathrm{\%}$ | xxxx | -0.1184 | -0.0044 | I | 54 | -0.1004 |
| 23 | C $\mathrm{H}^{\prime}{ }^{\prime} \mathrm{DX} 40 \%$ | xx | 0.0829 | 0.0903 | L | 55 | -0.0012 |
| 24 | C $H^{\prime}{ }_{0} \mathrm{DX} 60 \%$ | xx | 0.0557 | 0.2859 | L | 55 | -0.0087 |
| 25 | $\mathrm{C}^{3} \mathrm{H}_{0}$ | xxxx | -0.0014 | -0.1149 | I | 47 | -0.0383 |
| 26 | $\mathrm{C}^{3} \mathrm{H}_{0} \mathrm{Be} 40 \%$ | xx | 0.1180 | 0.2248 | L | 47 | 0.0202 |
| 27 | $\mathrm{C}^{3} \mathrm{H}_{0} \mathrm{Be} 60 \%$ | xx | 0.0901 | 0.3221 | L | 55 | 0.0094 |
| 28 | C $H_{\mathrm{R}}$ | xxxx | -0.3258 | -0.3704 | I | 56 | 0.2507 |
| 29 | C $H_{\mathrm{R}} \mathrm{DX} 40 \%$ | xx | -0.3591 | -0.5171 | L | 57 | -0.2381 |
| 30 | C $\mathrm{H}_{\mathbf{R}}$ DX60\% | xx | -0.4791 | -0.3493 | L | 57 | -0.2923 |
| 31 | $\mathrm{C} \Delta$ | xxxx | 0.3646 | -0.0299 | I | 31 | 0.1822 |
| 32 | C $\triangle$ DX40\% | xx | 0.4424 | -0.1229 | L | 47 | 0.1700 |
| 33 | C $\triangle$ DX60\% | xx | 0.4788 | -0.3851 | L | 47 | 0.1800 |
| 34 | C $\mathrm{H}_{\mathrm{GF}}$ | xx | -0.1807 | -0.0827 | L | 28 | -0.1381 |
| 35 | C $\lg a_{\mathrm{H}}$ | xx | 0.7801 | -0.0768 | L | 37 | 0.4172 |
| 36 | С $M_{\text {c }}$ | xxxxx | -0.0265 | 0.2998 | H | 27 | 0.0156 |
| 37 | $\mathrm{Cl} H_{0}$ | xxxxx | 0.0061 | -0.1246 | H | 45 | -0.0101 |
| 38 | Cl $H_{0} \mathrm{Et} 50 \%$ | xxxx | -0.0227 | -0.1354 | M | 54 | -0.0557 |
| 39 | Cl $H_{0} \mathrm{AN} 10 \%$ | x | -0.8124 | -0.0916 | M | 58 |  |
| 40 | Cl $H_{0} \mathrm{ANN} 30 \%$ | x | -0.1667 | 0.2748 | M | 58 |  |
| 41 | Cl $H_{0} \mathrm{AN} 50 \%$ | x | 0.2947 | 0.5431 | M | 58 |  |
| 42 | Cl $H_{0} \mathrm{AN} 80 \%$ | x | 0.0252 | 0.0026 | M | 58 |  |
| 43 | $\mathrm{Cl} H_{\mathrm{R}}$ | xxxxx | -0.1280 | -0.1267 | H | 2 | -0.0955 |
| 44 | $\mathrm{Cl} H_{\text {A }}$ | xxxxx | 0.0172 | -0.1581 | H | 45 | -0.0087 |
| 45 | $\mathrm{Cl} \Delta$ | xxxxx | 0.1945 | -0.0593 | H | 31 | 0.1195 |
| 46 | B ${ }^{3} H_{0}$ | xxxxx | 0.0185 | -0.1273 | H | 59 | -0.0043 |
| 47 | B $\Delta$ | xxxxx | 0.2203 | 0.0407 | H | 47 | 0.1503 |
| 48 | $\mathrm{J}^{3} \mathrm{H}_{0}$ | xxx | 0.2591 | -0.7743 | K | 60 | 0.0727 |
| 49 | J $\Delta$ | xxx | 0.5409 | 0.6116 | K | 47 | 0.1253 |
| 50 | FS $H_{\text {GF }}$ | xxxx | -0.2781 | 0.1160 | I | 61 | -0.1528 |
| 51 | $\mathrm{CF}_{3} H_{\text {GF }}$ | xxxx | 0.0189 | 0.3949 | I | 61 | 0.0482 |

${ }^{a}$ See footnotes to Table $1 .{ }^{b} \mathrm{x}, 2.5 \mathrm{M}$; xx, 4.0 M ; xxx, 5.5 M ; xxxx, 7.0 M ; xxxxx, $10.0 \mathrm{M} .{ }^{c}$ Submatrices $\mathrm{H}-\mathrm{M}$.
ranges of acid concentration have been used, $2.5,4.0$, $5.5,7.0$, and 10.0 M in 0.5 M intervals and are indicated in Table 3 for each acidity function by M, L, K, I, and H, respectively.

Mathematical Description of C.v.a.-C.v.a. ${ }^{13}$ can be used to estimate the number of independent orthogonal vectors contributing to the total variation observed in the data set. In the case of values of $H_{i}$ taken at $n$ concentrations, $i=1,2 \ldots n$ constitute one row of an $n$-column data vector. For $m$ acidity functions the $m$ vectors can be arranged to form an $m$. $n$ data matrix. The characteristic vectors $V_{m, l}$ are uniquely determined for a matrix of acidity functions and apply to all data
vectors. Characterisitc vectors form a set of $i=1,2 \ldots n$ numbers.

Acidity function may be represented by equation (2) where
$H_{m, l}=\bar{H}_{m, l}+S_{m, 1} V_{1, \iota}+S_{m, 2} V_{2 . l}+\ldots+S_{m, k} V_{k . l}$
the $S$ values are the amounts of the characteristic vectors which must be added to the mean acidity function vector $\bar{H}_{m, t}$ in order to obtain the sample vector. The parameter $S$ is specific to the acidity function. The number of characteristic vectors required to represent all the variation among the data set will be equal to or less than $n$ and in general is much less than $n$. The number of characteristic vectors is smaller, the closer the

Table 4. Results of c.v.a. calculation for submatrices $\mathbf{H}-\mathrm{M}$ (concentration mode)

|  |  | otal ation |  |  | Concentration (mol) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| matrix | $V_{1}$ | $V_{1}+V_{2}$ | $N^{\text {a }}$ |  | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 |
| $\begin{aligned} & 10 . \mathrm{m} \\ & \mathrm{H} \end{aligned}$ | 99.07 | 99.87 | 24 | $V$ | -0.206 | 1.104 | 2.347 | 3.419 | 4.507 | 5.551 | 6.594 | 7.702 | 8.878 |
|  |  |  |  | $V_{2}$ | 0.415 | 0.729 | 1.049 | 1.237 | 1.284 | 1.309 | 1289 | 1.216 | 1.150 |
|  |  |  |  | $-\overline{\boldsymbol{H}}$ | -0.108 | 0.191 | 0.401 | 0.590 | 0.777 | 0.954 | 1128 | 1.318 | 1.505 |
| $\begin{aligned} & \text { 7.0м } \\ & \text { I } \end{aligned}$ | 98.28 | 99.75 | 8 | $V_{1}$ | -0.146 | 1.223 | 2.269 | 3.143 | 4.013 | 4.954 | 5.948 | 6.900 | 7.877 |
|  |  |  |  | $V_{2}$ | 1.756 | 1.488 | 1.329 | 1.344 | 1.117 | 0.926 | 0.677 | 0.407 | 0.187 |
|  |  |  |  | $-\bar{H}$ | -0.234 | 0.168 | 0.435 | 0.672 | 0.905 | 1.157 | 1.456 | 1.735 | 2.004 |
| $\begin{aligned} & 5.5 \mathrm{M} \\ & \mathrm{~K} \end{aligned}$ | 98.03 | 99.99 | 3 | $V_{1}$ | -2.267 | -1.794 | -1.292 | -0.844 | -0.456 | 0.012 | 0.684 | 1.366 | 2.017 |
|  |  |  |  | $V_{2}$ | 0.095 | 0.235 | 0.279 | 0.306 | 0.338 | 0.344 | 0.303 | 0.242 | 0.167 |
|  |  |  |  | $-\bar{H}$ | -1.143 | -0.827 | -0.551 | -0.325 | -0.130 | 0.097 | 0.402 | 0.707 | 0.993 |
| $\begin{aligned} & 4.0 \mathrm{~m} \\ & \mathrm{~L} \end{aligned}$ | 93.42 | 99.56 | 11 | $V_{1}$ | -1.924 | -0.164 | 1.017 | 2.173 | 3.262 | 4.355 | 5.424 | 6.507 |  |
|  |  |  |  | $V_{2}$ | 1.838 | 1.288 | 1.085 | 0.812 | 0.511 | 0.267 | -0.035 | -0.271 |  |
|  |  |  |  | $-\bar{H}$ | -0.861 | -0.403 | 0.082 | 0.533 | 0.963 | 1.383 | 1.812 | 2.221 |  |
| $\begin{aligned} & 2.5 \mathrm{~m} \\ & \mathrm{M} \end{aligned}$ | 70.35 | 98.20 | 4 | $V_{1}$ | 0.782 | 0.661 | 0.470 | 0.228 | -0.036 |  |  |  |  |
|  |  |  |  | $V_{2}$ | -0.130 | -0.044 | 0.146 | 0.368 | 0.589 |  |  |  |  |
|  |  |  |  | $-\bar{H}$ | -1.035 | -0.530 | -0.133 | 0.207 | 0.517 |  |  |  |  |


|  |  | otal | $N^{\text {a }}$ |  | Concentration (mol) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| matrix | $V_{1}$ | $V_{1}+V_{2}$ |  |  | 5.0 | 5.5 | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 | 9.0 | 10.0 |
| ${ }_{\mathbf{H}}^{10.0 \mathrm{~m}}$ | 99.07 | $99.87$ | 24 | $V_{1}$ | 10.06 | 11.30 | 12.55 | 13.76 | 15.06 | 16.39 | 17.69 | 20.45 | 23.37 |
|  |  |  |  | $V_{2}$ | 1.053 | 0.989 | 0.775 | 0.493 | 0.217 | -0.120 | -0.505 | -1.318 | -2.225 |
|  |  |  |  | $-\bar{H}$ | 1.718 | 1.896 | 2.112 | 2.322 | 2.543 | 2.769 | 2.995 | 3.488 | 4.002 |
| $7.0 \mathrm{~m}$ | 98.28 | 99.75 | 8 | $V_{1}$ | 8.992 | 10.06 | 11.18 | 12.60 | 13.91 |  |  |  |  |
|  |  |  |  | $V_{2}$ | 0.044 | -0.214 | -0.490 | -0.660 | -0.765 |  |  |  |  |
|  |  |  |  | $-\bar{H}$ | 2.273 | 2.549 | 2.843 | 3.252 | 3.570 |  |  |  |  |
| $\begin{aligned} & 5.5 \mathrm{~m} \\ & \mathrm{~K} \end{aligned}$ | 98.03 | 99.99 | 3 | $V_{1}$ | 2.556 | 3.024 |  |  |  |  |  |  |  |
|  |  |  |  | $V_{2}$ | 0.115 | 0.078 |  |  |  |  |  |  |  |
|  |  |  |  | $-\bar{H}$ | 1.233 | 1.443 |  |  |  |  |  |  |  |
| $\begin{aligned} & 4.0 \mathrm{~m} \\ & \mathrm{~L} \end{aligned}$ | 93.42 | 99.56 | 11 | $V_{1}$ |  |  |  |  |  |  |  |  |  |
|  |  |  |  | $-\overline{V_{2}}$ |  |  |  |  |  |  |  |  |  |
| 2.5m | 70.35 | 98.20 | 4 |  |  |  |  |  |  |  |  |  |  |
| M |  |  |  | $V_{2}$ |  |  |  |  |  |  |  |  |  |

${ }^{a}$ Number of acidity functions.
similarity between the acidity functions. The importance of a particular vector is measured as percentage of the total variability, $\% \mathrm{TV}$, explained by this vector.

A mathematical method for estimating the number of characteristic vectors and their values has been developed for complete matrices only. The method allows us to calculate the first reconstituted data matrix using only the first characteristic vector $V_{1, \iota}$ and the percentage of total variability of the original data described by this matrix. The calculation is continued until the percentage of total variability reaches $100 \%$ or with the desired and declared number of characteristic vectors.

The mathematical model described by equation (2) is seen to have the multiparameter form of a linear free energy relationship (l.f.e.r.) and should be valid for the following conditions; (1) the acidity function (index $m$ ) has some degree of similarity, (2) the variables of concentration are related to this similarity and have continuous properties in the range $i=1,2 \ldots$
$n$. Both conditions are well fulfilled in acidity function data set.

## Results and Discussion

Tables 1-4 present the most significant results of the c.v.a. calculation for 142 acidity functions of 15 strong acids as variables of percentage and molarity divided into 12 submatrices. Acidity functions available for the $95 \%$ parameters are grouped into two submatrices A and B because of much better results for separate treatments. The number of acidity functions in each submatrix is given by $N$; the acid and acidity function used in each submatrix can be found from the data in columns 2 and 5 of Tables 1 and 3.

One variable suffices to explain $>98 \%$ of the variance for several acidity functions and acids of various chemical properties, except submatrices $L$ and $M$. An increase in the number of variables to two will improve this explanation to

Table 5. Parameters of linear and/or potential regression of $V_{1, \iota}$ and ranges of acid concentration

| $5-35 \%$ or 0.5-5.0M |  |  |  |  | $40-95 \%$ or 6-10M |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Submatrix | $V_{1}=b C+a$ | $r$ | $s$ | $n$ | $V_{1}=a C^{b}$ | $r$ | $s$ | $n$ |
| A | $0.1726 C-0.8419$ | 0.9986 | 0.0331 | 7 | $0.0148 C^{1.6361}$ | 0.9992 | 0.0773 | 12 |
| B | $0.1566 C-0.8036$ | 0.9951 | 0.0529 | 7 | $0.0413 C^{1.3282}$ | 0.9963 | 0.1021 | 12 |
| C | $0.1396 C+0.4473$ | 0.9952 | 0.0495 | 7 | $0.0431 C^{1.3570}$ | 0.9971 | 0.1320 | 11 |
| D | $0.2587 C+2.2788$ | 0.9998 | 0.0131 | 7 | $0.0274 C^{1.5540}$ | 0.9998 | 0.0517 | 9 |
| E | $0.1790 C-1.0436$ | 0.9998 | 0.0342 | 7 | $0.0103 C^{1.7347}$ | 0.9934 | 0.1328 | 7 |
| F | 0.4326 C-1.7906 | 0.9988 | 0.0733 | 7 | $0.0466 C^{1.5815}$ | 0.9979 | 0.0406 | 4 |
| G | $0.2265 C-0.5327$ | 0.9970 | 0.0435 | 7 |  |  |  |  |
| H | $2.2517 \mathrm{M}-1.2503$ | 0.9992 | 0.0548 | 13 | $1.3739 \mathrm{~m}^{1.2298}$ | 0.9999 | 0.0136 | 5 |
| I | $1.8677 \mathrm{~m}-0.6170$ | 0.9885 | 0.0572 | 10 |  |  |  |  |
| K | $1.0990 \mathrm{~m}-3.0428$ | 0.9936 | 0.0715 | 10 |  |  |  |  |
| L | $2.2114 \mathrm{~m}-2.3057$ | 0.9997 | 0.0237 | 8 |  |  |  |  |
| M | $-0.4124 \mathrm{~m}+1.0394$ | 0.9803 | 0.0126 | 5 |  |  |  |  |

$r$, Correlation coefficient; $s$, standard deviation; $n$, number of observations; $C$, percentage; m molarity.


Figure 1. Plot of characteristic vectors $V_{2}$ against $V_{1}$ for submatrix A as a function of concentration
$>99.5 \%$ of the experimental variance in the data set except for submatrix M. The very low result for submatrix $M$ is associated with random scattering, poor representation of the acidity functions, and very narrow concentration range ( 2.5 m hydrochloric acid is $c a .10 \%$ ). The submatrix M is intermediate between the lower and higher concentration ranges.

The 'characteristic vectors' $V_{1, i}$ and $V_{2, i}$ are the only significant eigenvalues. Their values were presented in Tables 2 and 4 for all submatrices as a function of concentration. $V_{1.1}$ and $V_{2,4}$ are presented by Figure 1 for submatrix A. The shape of $V_{1, t}$ is in general similar to the graph of acidity function versus percentage of acid concentration. The shape of all $V_{1, j}$ vectors (submatrices $\mathrm{B}-\mathrm{L}$ ) are very similar and consist

Table 6. Linear regression equations for $V_{1}$ characteristic vectors

| Regression equation | $r$ | $s$ | $n$ |
| :---: | :---: | :---: | :---: |
| $V_{\mathbf{B}}=0.9107 V_{\mathrm{A}}+0.6736$ | 0.9865 | 0.1214 | 19 |
| $V_{\mathrm{C}}=0.7824 V_{\mathrm{A}}+1.4087$ | 0.9944 | 0.1440 | 18 |
| $V_{\mathrm{D}}=1.3393 V_{\mathrm{A}}-0.4183$ | 0.9971 | 0.0469 | 16 |
| $V_{\mathrm{B}}=1.0817 V_{\mathrm{A}}-0.3451$ | 0.9933 | 0.0935 | 14 |
| $V_{\mathbf{F}}=2.5381 V_{\mathrm{A}}+0.2138$ | 0.9989 | 0.1099 | 11 |
| $V_{\mathbf{G}}=1.2911 V_{\mathrm{A}}+0.6100$ | 0.9991 | 0.0432 | 7 |
|  |  |  |  |
| $V_{\mathbf{I}}=0.8771 V_{\mathrm{H}}+0.1176$ | 0.9997 | 0.0347 | 12 |
| $V_{\mathrm{K}}=0.4785 V_{\mathrm{H}}-2.3381$ | 0.9972 | 0.0469 | 11 |
| $V_{\mathbf{L}}=1.0488 V_{\mathrm{H}}-1.3196$ | 0.9998 | 0.0155 | 7 |
| $V_{\mathrm{M}}=-0.1747 V_{\mathrm{H}}+0.8114$ | 0.9664 | 0.0188 | 5 |

of two parts, linear in the low concentration range (5-35\% and $0.5-5.0 \mathrm{~m}$ ) and curved at higher concentrations. The curved part of the graph can be represented by equation (3)

$$
\begin{equation*}
V_{1, t}=a C^{b} \tag{3}
\end{equation*}
$$

where $a$ and $b$ represent regression parameters and $C$ is the concentration. The parameters for linear and potential regression are presented in Table 5.

The first characteristic vectors $V_{1, i}$ calculated for various submatrices are linearly related to each other. The $V_{1, l}$ for submatrices A (percentage mode) and H (molarity mode) were chosen arbitrarily as typical values for submatrices B-G and I-M, respectively, and regression parameters are shown in Table 6. As a rule, the correlation coefficients are very high in all cases.

Regression parameters between $V_{\mathrm{A}}$ and some acidity functions for sulphuric acid are given in Table 7 and show a good linear relation. A very good linear relation also exists between $\log a_{\mathbf{H}^{+*}}$, the proton activities of various acids, as indicated by the data in Table 7. In the literature $\log a_{\mathrm{H}}{ }^{+*}$ is considered to be a better operational scale of acidity in concentrated acids ${ }^{18}$ than any other acidity function thus far used.

A two-term equation (4) has been found for describing the various acidity functions in organic and inorganic acids using

$$
\begin{equation*}
H_{m, t}=\bar{H}_{m, t}+V_{1, t} S_{m, t}+V_{2, t} S_{m, 2} \tag{4}
\end{equation*}
$$

numerical values from Tables 1-4. The last term in equation (4) could be neglected in acidity function reconstitution if its value does not exceed the experimental error, normally $\pm 0.05$

Table 7. Linear regression equation between various acidity functions and reference value $V_{\mathrm{A}}$

$$
\begin{array}{rrr}
\text { S } H_{0}= & -0.3806 V_{\mathrm{A}}+0.0041 & r 0.9995 \\
\mathrm{~S} H_{\mathrm{x}} & =0.3432 V_{\mathrm{A}}-0.4353 & r 0.9862 \\
\mathrm{~S} H_{\mathrm{MC}}= & -0.3656 V_{\mathrm{A}}+0.0167 & r 0.9946 \\
\mathrm{~S} 1 \mathrm{~g} C_{\mathrm{H}^{+}}= & 0.5308 V_{\mathrm{A}}+0.4588 & r 0.9594 \\
\text { S } 1 \mathrm{~g} a_{\mathrm{H}^{+}}= & 0.6742 V_{\mathrm{A}}+0.0612 & r 0.9994 \\
\mathrm{C} 1 \mathrm{~g} a_{\mathrm{H}^{+}}= & 0.8159 V_{\mathrm{A}}-0.1700 & r 0.9972 \\
\text { P } 1 \mathrm{~g} a_{\mathrm{H}^{+}}= & 0.4609 V_{\mathrm{A}}-0.5368 & r 0.9758 \\
\text { HP } 1 \mathrm{~g} a_{\mathrm{H}^{+}}= & 0.2450 V_{\mathrm{A}}-0.2519 & r 0.9929 \\
\text { TS } 1 \mathrm{~g} a_{\mathrm{H}^{+}}= & 0.3963 V_{\mathrm{A}}-0.5884 & r 0.9797 \\
\text { MS } 1 \mathrm{~g} a_{\mathrm{H}^{+}}= & 0.3523 V_{\mathrm{A}}+0.1835 & r 0.9990
\end{array}
$$

unit. The magnitude of $V_{2 . t}$ is not very large and in most submatrices at low concentration is constant or growing slowly; at higher concentration the magnitude of $V_{2, i}$ is increasing and in few cases is less than -1.5 .

In our opinion $V_{2.1}$ is associated with failure of acidity function determination procedure and experimental errors. In the method used, ionization ratios are not always perfectly linear and parallel and introduce cumulative errors of uncertain magnitude. ${ }^{37}$ Multiplication of $V_{2, \iota}$ by $S_{2 . m}$ yield the contribution of all errors to the acidity function. This contribution depends on concentration and could be neglected under condition $V_{2.4} S_{m .2}<0.05$. Thus equation (5) simplifies to a one-parameter equation. Consequently there appears to be no

$$
\begin{equation*}
H_{m, l}=\bar{H}_{m, i}+V_{1, i} S_{m, 2} \tag{5}
\end{equation*}
$$

more than one type of variable which produce a significant and truly independent difference in the acidity function data set. In other words, any theory of acidity functions which explains these properties in terms of more than one adjustable parameter seems likely to contain redundant information. The existence of only one significant characteristic vector indicates the close similarity between acidity functions in all submatrices. The reconstituted acidity function set using the parameters of equation (5) will be the best and will be error free.

The question arises: what is a chemical significance of $V_{1,1}$ and $S_{m, 1}$ ? The specific pattern of $V_{1, i}$ in all submatrices, the possibility of a mathematical description (see Table 5) as a function of concentration, the linear relation for all $V_{1, t}$ values in submatrices $\mathrm{A}-\mathrm{G}$ and $\mathrm{H}-\mathrm{L}$, and the independence of the nature of the acid lead to the conclusion that the first characteristic vector is itself a statistically universal measure of acidity. The numerical values of $V_{1, i}$ are not fixed because they depend on the initial data set. $V_{1, i}$ calculated for the concentration range $5-95 \%$ (submatrix A) was chosen arbitrarily as the reference variable for all submatrices in the percentage mode. Its numerical values can be used as the universal acidity measure instead of $M_{\mathrm{c}}{ }^{8}{ }^{8} X,{ }^{10}$ or $\log a_{\mathrm{H}}{ }^{+*} .^{37}$ The advantage of $V_{1, A}$ and $V_{1, H}$ is their independence of the nature of the acid. They depend on the concentration, however.

The rationalization of $S_{m, 1}$ and $S_{m, 2}$ values with respect to a particular type of acidity function (e.g. $H_{0}, H_{\mathrm{R}}, H_{\mathrm{A}}$ ), acid, and concentration is very difficult due to the fact that these parameters were calculated for separate submatrices. However some existing trends are in good agreements with the results of another approach. That approach was derived from the results of calculations in which 112 or 102 acidity functions were used in acidity ranges $5-40$ or $5-55 \%$, respectively. The $S_{1}-S_{2}$ relation for all acids and the 102 acidity functions is presented in Figure 2.

The points for perchloric and sulphuric acids are widely distributed along the $S_{1}$ axis together with the narrow distribution along the $S_{2}$ axis $(+0.05$ and -0.06$)$. The similar


Figure 2. Plot of $S_{2}$ against $S_{1}$ for 102 acidity functions in the 5-55\% concentration range
data for hydrochloric acid (in the acidity range 5- $40 \%$ ) are distributed in a similar way along the $S_{1}$ axis and in the -0.08 to -0.12 range of the $S_{2}$ axis. Data for nitric acid are scanty and are distributed below these for other strong acids; they yield a linear relationship of positive slope. The picture is similar for phosphoric acid; however $S_{2}$ values are positive and the slope is negative. Points for toluene-p-sulphonic acid are widely distributed along the $S_{2}$ axis together with a narrow distribution along the $S_{1}$ axis. Points for other acids are very limited. Data for selenic acid and methanesulphonic acid fall in the area for sulphuric acid. Data for hydrofluoric acid, trifluoroacetic acid, and phosphonic acid overlap with those for phosphoric acid.

Careful inspection of the $S_{1 . m}$ data for sulphuric acid from the set of 102 acidity functions in the concentration range $5-55 \%$ leads to the conclusion that numerous acidity functions have identical figures. For example $H_{0}{ }^{3}, H_{0}^{\prime}$, and $H^{\prime \prime \prime}{ }_{0}$ have $S_{1, m}-0.036$ and $H_{1}$ and $H_{\mathrm{T}}$ have $S_{1, m}-0.079$. Frequently $S_{1, m}$ values for various acidity functions are very close to each other, e.g. $\Delta$ and $\log C_{\mathrm{H}}, H_{0}{ }^{3}$ and $H_{0}, H_{\mathrm{GF}}$ and $\mathrm{H}_{\mathrm{C}}$. The conclusion is that these acidity functions have much the same nature and follow the cancellation assumption. ${ }^{19}$ Thus the number of original acidity functions could be reduced, though not to five as suggested by Carpentier. ${ }^{61}$

Similar trends could be observed for the $S_{m .1}$ and $S_{m .2}$ data for acidity functions in water-organic mixtures as variables of the molar concentration. That approach was derived as a result of calculating all existing data in the concentration range $0.5-4.0 \mathrm{~m}$. The $S_{1}-S_{2}$ relationship for 44 acidity functions as variables of the molar concentration is presented by Figure 3.

Points are distributed mainly along the $S_{1}$ axis together with a narrow distribution along the $S_{2}$ axis. Limited numbers of points strongly deviating along the $S_{2}$ axis represent acidity functions in mixed solvents, water-dioxan, water-2-butoxyethanol, and ocasionally water-ethanol. The number of data is not very large, however; Figure 4 shows the relation be-


Figure 3. Plot of $S_{2}$ against $S_{1}$ for acidity functions in $0.5-4.0 \mathrm{~m}$ concentration range


Figure 4. $S_{2}$ values for selected acidity functions in water-organic solvents as a function of solvent parameter $Y$
tween $S_{m, 2}$ and the polarity of solvents as measured by the $Y$ parameter. ${ }^{62}$ The relationship is not very far from linear. The importance of $S_{m .2}$ is not very large due to the small value of $V_{2 . t}$. However in the $H_{\mathrm{GF}}$ acidity function in sulphuric acid and the $H_{\mathrm{R}}$ and $\delta$ acidity functions in perchloric acid, the contribution of $S_{m .2} V_{2 . t}$ is significant and depends on the polarity.

A linear relation of $S_{m, 1}$ for various acids with $S_{m, 1}$ for sulphuric acid as reference parameter suggests that the nature of acidity functions of identical type is independent of the acid, and activity coefficient terms are related to each other. The proportionality factor is simply the slope of the lines in Figures 5 and 6. This slope reflects the nature of the acid and the decrease in $\mathrm{p} K_{\mathrm{a}}$ of acids in a qualitative way. It is desirable to focus attention on this problem in future. A slope $>1$ indicate a stronger acid than sulphuric or that the numerical value of the desired acidity function at the same concentration is more negative than in sulphuric acid. A slope $<1$ indicates the opposite.

The results of our method can be used for prediction pur-


Figure 5. Plot of $S_{m .1}$ for various acids and acidity functions in the $5-55 \%$ concentration range against $S_{m, 1}$ for sulphuric acid as reference parameter (percentage mode)


Figure 6. Plot of $S_{m .1}$ for various acids and acidity functions in the concentration range $0.5-4.0 \mathrm{~m}$ against $S_{m, 1}$ for sulphuric acid as reference parameter (concentration mode) (see Table 4 for H and L)
poses. First, the data collected in Tables 1-4 are sufficient to provide the best values of any acidity function used in calculations. Also, an unknown acidity function could be predicted for any acid studied with the condition, however, that the relationship of $S_{m, 1}$ and $S_{m, 1}$ (for sulphuric acid as reference parameter) is defined. Thus new acidity functions for hydrochloric, nitric, methanesulphonic, and phosphoric acids are accessible.
The application of the statistically universal measure of acidity, $V_{1 . \iota}$ to protonation phenomena and kinetic problems in acid catalysed reactions will be discussed in future papers.

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